

WHAT IS CLAIMED IS:

1. An arithmetic performance attribution method for determining portfolio performance, relative to a benchmark, over multiple time periods  $t$ , where  $t$  varies from 1 to  $T$ , comprising the steps of:

- 5 (a) determining coefficients  $(A + \alpha_t)$ , where the values  $\alpha_t$  are defined as

$$\alpha_t = \left[ \frac{R - \bar{R} - A \sum_{k=1}^T (R_k - \bar{R}_k)}{\sum_{k=1}^T (R_k - \bar{R}_k)^2} \right] (R_t - \bar{R}_t),$$

where  $A$  has any predetermined value,  $R_t$  is a portfolio return for period  $t$ ,  $\bar{R}_t$  is a benchmark return for period  $t$ ,  $R$  is determined by

$$R = \left[ \prod_{t=1}^T (1 + R_t) \right] - 1,$$

- 10 and  $\bar{R}$  is determined by

$$\bar{R} = \left[ \prod_{t=1}^T (1 + \bar{R}_t) \right] - 1;$$

and

- (b) determining the portfolio performance as

$$R - \bar{R} = \sum_{t=1}^T (A + \alpha_t)(R_t - \bar{R}_t).$$

15

2. The method of claim 1, wherein  $A$  is

$$A = \frac{1}{T} \left[ \frac{(R - \bar{R})}{(1 + R)^{1/T} - (1 + \bar{R})^{1/T}} \right], \text{ where } R \neq \bar{R},$$

or for the special case  $R = \bar{R}$ :

$$A = (1 + R)^{(T-1)/T}.$$

20

3. The method of claim 1, wherein  $A = 1$ .

4. The method of claim 1, wherein step (b) is performed by determining the portfolio performance as

$$25 \quad R - \bar{R} = \sum_{t=1}^T \sum_{i=1}^N (A + \alpha_t)(I_{it}^A + S_{it}^A),$$

where  $I_{it}^A$  is an issue selection for sector  $i$  and period  $t$ , and  $S_{it}^A$  is a sector selection for sector  $i$  and period  $t$ .

5            5. A computer system, comprising:

a processor programmed to perform an arithmetic performance attribution computation to determine portfolio performance, relative to a benchmark, over multiple time periods  $t$ , where  $t$  varies from 1 to  $T$ , by determining coefficients  $(A + \alpha_t)$ , where the values  $\alpha_t$  are defined as

$$10 \quad \alpha_t = \left[ \frac{R - \bar{R} - A \sum_{k=1}^T (R_k - \bar{R}_k)}{\sum_{k=1}^T (R_k - \bar{R}_k)^2} \right] (R_t - \bar{R}_t),$$

where  $A$  has any predetermined value,  $R_t$  is a portfolio return for period  $t$ ,  $\bar{R}_t$  is a benchmark return for period  $t$ ,  $R$  is determined by

$$R = \left[ \prod_{t=1}^T (1 + R_t) \right] - 1,$$

and  $\bar{R}$  is determined by

$$15 \quad \bar{R} = \left[ \prod_{t=1}^T (1 + \bar{R}_t) \right] - 1;$$

and determining the portfolio relative performance as

$$R - \bar{R} = \sum_{t=1}^T (A + \alpha_t)(R_t - \bar{R}_t); \text{ and}$$

a display device coupled to the processor for displaying a result of the arithmetic performance attribution computation.

20

6. A computer readable medium which stores code for programming a processor to perform an arithmetic performance attribution computation to determine portfolio performance, relative to a benchmark, over multiple time periods  $t$ , where  $t$  varies from 1 to  $T$ , by determining coefficients  $(A + \alpha_t)$ , where the values  $\alpha_t$  are defined as

$$25 \quad \alpha_t = \left[ \frac{R - \bar{R} - A \sum_{k=1}^T (R_k - \bar{R}_k)}{\sum_{k=1}^T (R_k - \bar{R}_k)^2} \right] (R_t - \bar{R}_t),$$

where  $A$  has any predetermined value,  $R_t$  is a portfolio return for period  $t$ ,  $\bar{R}_t$  is a benchmark return for period  $t$ ,  $R$  is determined by

$$R = \left[ \prod_{t=1}^T (1 + R_t) \right] - 1,$$

and  $\bar{R}$  is determined by

$$\bar{R} = \left[ \prod_{t=1}^T (1 + \bar{R}_t) \right] - 1;$$

and determining the portfolio relative performance as  $R - \bar{R} = \sum_{t=1}^T (A + \alpha_t)(R_t - \bar{R}_t)$ .

7. A geometric performance attribution method for determining portfolio performance, relative to a benchmark, over multiple time periods  $t$ , where  $t$  varies from 1 to  $T$ , comprising the steps of:

determining attribution effects for issue selection  $(1 + I_{it}^G)$  given by

$$1 + I_{it}^G = \frac{1 + w_{it} r_{it}}{1 + w_{it} \bar{r}_{it}} \Gamma_{it}^I,$$

and determining attribution effects for sector selection  $(1 + S_{it}^G)$  given by

$$1 + S_{it}^G = \left( \frac{1 + w_{it} \bar{r}_{it}}{1 + \bar{w}_{it} \bar{r}_{it}} \right) \left( \frac{1 + \bar{w}_{it} \bar{R}_t}{1 + w_{it} \bar{R}_t} \right) \Gamma_{it}^S,$$

where  $r_{jt}$  is a portfolio return for sector  $j$  for period  $t$ ,  $\bar{r}_{jt}$  is a benchmark return for sector  $j$  for period  $t$ ,  $w_{jt}$  is a weight for  $r_{jt}$ ,  $\bar{w}_{jt}$  is a weight for  $\bar{r}_{jt}$ ,  $R$  is determined by

$$R = \left[ \prod_{t=1}^T (1 + R_t) \right] - 1$$

and  $\bar{R}$  is determined by

$$\bar{R} = \left[ \prod_{t=1}^T (1 + \bar{R}_t) \right] - 1;$$

and determining the portfolio performance as

$$\frac{1 + R}{1 + \bar{R}} = \prod_{t=1}^T \prod_{i=1}^N (1 + I_{it}^G)(1 + S_{it}^G).$$

5

$$\Gamma_l^S = \left[ \frac{1 + \tilde{R}_l}{1 + \bar{R}_l} \prod_{j=1}^N \left( \frac{1 + \bar{w}_{jl} \bar{r}_{jl}}{1 + w_{jl} \bar{r}_{jl}} \right) \left( \frac{1 + w_{jl} \bar{R}_l}{1 + \bar{w}_{jl} \bar{R}_l} \right) \right]^{1/N}.$$

10

$$\Gamma_t^I = \Gamma_t^S = \Gamma_t = \left[ \left( \frac{1+R_t}{1+\bar{R}_t} \right) \prod_{j=1}^N \frac{(1+\bar{w}_{jt}\bar{r}_{jt})(1+w_{jt}\bar{R}_t)}{(1+w_{jt}r_{jt})(1+\bar{w}_{jt}\bar{R}_t)} \right]^{\frac{1}{2N}}.$$

a processor programmed to perform a geometric performance attribution computation to determine portfolio performance, relative to a benchmark, over multiple time periods  $t$ , where  $t$  varies from 1 to  $T$ , by determining attribution effects for issue selection  $(1 + I_{it}^G)$  given by

$$1 + I_{ii}^G = \frac{1 + w_{ii} r_{ii}}{1 + w_{ii} \bar{r}_{ii}} \Gamma_i^I,$$

15 and determining attribution effects for sector selection  $(1 + S_{ij}^G)$  given by

$$1 + S_{ii}^G = \left( \frac{1 + w_{ii} \bar{r}_{ii}}{1 + \bar{w}_{ii} \bar{r}_{ii}} \right) \left( \frac{1 + \bar{w}_{ii} \bar{R}_i}{1 + w_{ii} \bar{R}_i} \right) \Gamma_i^S,$$

where  $r_{jt}$  is a portfolio return for sector  $j$  for period  $t$ ,  $\bar{r}_{jt}$  is a benchmark return for sector  $j$  for period  $t$ ,  $w_{jt}$  is a weight for  $r_{jt}$ ,  $\bar{w}_{jt}$  is a weight for  $\bar{r}_{jt}$ ,  $R$  is determined by

$$R = [\prod_{i=1}^T (1 + R_i)] - 1$$

and  $\bar{R}$  is determined by

$$\bar{R} = [\prod_{t=1}^T (1 + \bar{R}_t)] - 1,$$

and determining the portfolio performance as

$$\frac{1+R}{1+\bar{R}} = \prod_{t=1}^T \prod_{i=1}^N (1+I_{it}^G)(1+S_{it}^G);$$

and

- 5 a display device coupled to the processor for displaying a result of the geometric performance attribution computation.

11. The system of claim 10, wherein the values of  $\Gamma_t^I$  are

$$\Gamma_t^I = \left[ \frac{1+R_t}{1+\bar{R}_t} \prod_{j=1}^N \left( \frac{1+w_{jt}\bar{r}_{jt}}{1+w_{jt}r_{jt}} \right) \right]^{1/N} \text{ and the values of } \Gamma_t^S \text{ are}$$

$$10 \quad \Gamma_t^S = \left[ \frac{1+\bar{R}_t}{1+\bar{R}_t} \prod_{j=1}^N \left( \frac{1+\bar{w}_{jt}\bar{r}_{jt}}{1+\bar{w}_{jt}\bar{r}_{jt}} \right) \left( \frac{1+w_{jt}\bar{R}_t}{1+\bar{w}_{jt}\bar{R}_t} \right) \right]^{1/N}.$$

12. A computer readable medium which stores code for programming a processor to perform a geometric performance attribution computation to determine portfolio performance, relative to a benchmark, over multiple time periods  $t$ , where  $t$  varies from 1

- 15 to  $T$ , by determining attribution effects for issue selection  $(1+I_{it}^G)$  given by

$$1+I_{it}^G = \frac{1+w_{it}r_{it}}{1+w_{it}\bar{r}_{it}} \Gamma_t^I,$$

and determining attribution effects for sector selection  $(1+S_{it}^G)$  given by

$$1+S_{it}^G = \left( \frac{1+w_{it}\bar{r}_{it}}{1+\bar{w}_{it}\bar{r}_{it}} \right) \left( \frac{1+\bar{w}_{it}\bar{R}_t}{1+\bar{w}_{it}\bar{R}_t} \right) \Gamma_t^S,$$

where  $r_{jt}$  is a portfolio return for sector  $j$  for period  $t$ ,  $\bar{r}_{jt}$  is a benchmark return for sector

- 20  $j$  for period  $t$ ,  $w_{jt}$  is a weight for  $r_{jt}$ ,  $\bar{w}_{jt}$  is a weight for  $\bar{r}_{jt}$ ,  $R$  is determined by

$$R = \left[ \prod_{t=1}^T (1+R_t) \right] - 1$$

and  $\bar{R}$  is determined by

$$\bar{R} = \left[ \prod_{t=1}^T (1+\bar{R}_t) \right] - 1; \text{ and determining the portfolio performance as}$$

$$\frac{1+R}{1+\bar{R}} = \prod_{t=1}^T \prod_{i=1}^N (1+I_{it}^G)(1+S_{it}^G).$$

13. The computer readable medium of claim 12, wherein the values of  $\Gamma_t^I$  are

$$\Gamma_t^I = \left[ \frac{1+R_t}{1+\bar{R}_t} \prod_{j=1}^N \left( \frac{1+w_{jt}\bar{r}_{jt}}{1+w_{jt}r_{jt}} \right) \right]^{1/N} \text{ and the values of } \Gamma_t^S \text{ are}$$

$$\Gamma_t^S = \left[ \frac{1+\bar{R}_t}{1+\bar{R}_t} \prod_{j=1}^N \left( \frac{1+\bar{w}_{jt}\bar{r}_{jt}}{1+\bar{w}_{jt}\bar{r}_{jt}} \right) \left( \frac{1+w_{jt}\bar{R}_t}{1+\bar{w}_{jt}\bar{R}_t} \right) \right]^{1/N}.$$

14. A geometric performance attribution method for determining portfolio performance, relative to a benchmark, over multiple time periods  $t$ , where  $t$  varies from 1 to  $T$ , comprising the steps of:

10 determining attribution effects  $1+Q_{ijt}^G$  given by

$$1+Q_{ijt}^G = \prod_k \left( \frac{1+a_{ijt}^k}{1+b_{ijt}^k} \right) \Gamma_{ijt}^k,$$

where  $\Gamma_{ijt}^k$  are corrective terms that satisfy the constraint  $\prod_{ij} (1+Q_{ijt}^G) = \frac{1+R_t}{1+\bar{R}_t}$ , each of

$a_{ijt}^k$  and  $b_{ijt}^k$  is a coefficient for attribution effect  $j$ , sector  $i$ , and period  $t$ , the coefficients  $a_{ijt}^k$  and  $b_{ijt}^k$  are obtained from arithmetic attribution effects  $Q_{ijt}^A = \sum_k a_{ijt}^k - \sum_k b_{ijt}^k$  which

15 correspond to the attribution effects  $1+Q_{ijt}^G$ ,  $R_t$  is a portfolio return for period  $t$ ,  $\bar{R}_t$  is a benchmark return for period  $t$ , where  $R$  is determined by

$$R = \left[ \prod_{t=1}^T (1+R_t) \right] - 1$$

and  $\bar{R}$  is determined by

$$\bar{R} = \left[ \prod_{t=1}^T (1+\bar{R}_t) \right] - 1; \text{ and}$$

20 determining the portfolio performance as

$$\frac{1+R}{1+\bar{R}} = \prod_{t=1}^T \prod_{i=1}^N \prod_{j=1}^M (1+Q_{ijt}^G).$$

15. The method of claim 14, wherein  $M = 2$ ,  $1 + Q_{it}^G$  are attribution effects for issue election given by  $1 + Q_{it}^G = \frac{1 + w_{it}r_{it}}{1 + w_{it}\bar{r}_{it}} \Gamma_t^I$ , and  $1 + Q_{it}^G$  are attribution effects for sector selection given by  $1 + Q_{it}^G = \left( \frac{1 + w_{it}\bar{r}_{it}}{1 + \bar{w}_{it}\bar{r}_{it}} \right) \left( \frac{1 + \bar{w}_{it}\bar{R}_t}{1 + w_{it}\bar{R}_t} \right) \Gamma_t^S$ ,

5 where  $r_{it}$  is a portfolio return for sector  $i$  for period  $t$ ,  $\bar{r}_{it}$  is a benchmark return for sector  $i$  for period  $t$ ,  $w_{it}$  is a weight for  $r_{it}$ ,  $\bar{w}_{it}$  is a weight for  $\bar{r}_{it}$ , the values of  $\Gamma_t^I$  are

$$\Gamma_t^I = \left[ \frac{1 + R_t}{1 + \bar{R}_t} \prod_{i=1}^N \left( \frac{1 + w_{it}\bar{r}_{it}}{1 + w_{it}r_{it}} \right) \right]^{1/N}, \text{ and}$$

$$\text{the values of } \Gamma_t^S \text{ are } \Gamma_t^S = \left[ \frac{1 + \bar{R}_t}{1 + \bar{R}_t} \prod_{i=1}^N \left( \frac{1 + \bar{w}_{it}\bar{r}_{it}}{1 + w_{it}\bar{r}_{it}} \right) \left( \frac{1 + w_{it}\bar{R}_t}{1 + \bar{w}_{it}\bar{R}_t} \right) \right]^{1/N}.$$

10 16. A computer system, comprising:

a processor programmed to perform a geometric performance attribution computation to determine portfolio performance, relative to a benchmark, over multiple time periods  $t$ , where  $t$  varies from 1 to  $T$ , by determining attribution effects  $1 + Q_{ijt}^G$  given by

$$15 \quad 1 + Q_{ijt}^G = \prod_k \left( \frac{1 + a_{ijt}^k}{1 + b_{ijt}^k} \right) \Gamma_{ijt}^k,$$

where  $\Gamma_{ijt}^k$  are corrective terms that satisfy the constraint  $\prod_{ij} (1 + Q_{ijt}^G) = \frac{1 + R_t}{1 + \bar{R}_t}$ , each of

$a_{ijt}^k$  and  $b_{ijt}^k$  is a coefficient for attribution effect  $j$ , sector  $i$ , and period  $t$ , the coefficients  $a_{ijt}^k$  and  $b_{ijt}^k$  are obtained from arithmetic attribution effects  $Q_{ijt}^A = \sum_k a_{ijt}^k - \sum_k b_{ijt}^k$  which

correspond to the attribution effects  $1 + Q_{ijt}^G$ ,  $R_t$  is a portfolio return for period  $t$ ,  $\bar{R}_t$  is a

20 benchmark return for period  $t$ ,  $R$  is determined by

$$R = \left[ \prod_{t=1}^T (1 + R_t) \right] - 1$$

and  $\bar{R}$  is determined by

$$\bar{R} = \left[ \prod_{t=1}^T (1 + \bar{R}_t) \right] - 1, \text{ and}$$

determining the portfolio performance as  $\frac{1+R}{1+\bar{R}} = \prod_{t=1}^T \prod_{i=1}^N \prod_{j=1}^M (1 + Q_{ijt}^G)$ ; and

5 a display device coupled to the processor for displaying a result of the geometric performance attribution computation.

17. The system of claim 16, wherein  $M = 2$ ,  $1 + Q_{it}^G$  are attribution effects for

issue election given by  $1 + Q_{it}^G = \frac{1 + w_{it} r_{it}}{1 + w_{it} \bar{r}_{it}} \Gamma_t^I$ , and  $1 + Q_{i2t}^G$  are attribution effects for

10 sector selection given by  $1 + Q_{i2t}^G = \left( \frac{1 + w_{it} \bar{r}_{it}}{1 + \bar{w}_{it} \bar{r}_{it}} \right) \left( \frac{1 + \bar{w}_{it} \bar{R}_t}{1 + w_{it} \bar{R}_t} \right) \Gamma_t^S$ ,

where  $r_{it}$  is a portfolio return for sector  $i$  for period  $t$ ,  $\bar{r}_{it}$  is a benchmark return for sector  $i$  for period  $t$ ,  $w_{it}$  is a weight for  $r_{it}$ ,  $\bar{w}_{it}$  is a weight for  $\bar{r}_{it}$ , the values of  $\Gamma_t^I$  are

$$\Gamma_t^I = \left[ \frac{1 + R_t}{1 + \bar{R}_t} \prod_{i=1}^N \left( \frac{1 + w_{it} \bar{r}_{it}}{1 + w_{it} r_{it}} \right) \right]^{1/N}, \text{ and}$$

the values of  $\Gamma_t^S$  are  $\Gamma_t^S = \left[ \frac{1 + \bar{R}_t}{1 + \bar{R}_t} \prod_{i=1}^N \left( \frac{1 + \bar{w}_{it} \bar{r}_{it}}{1 + w_{it} \bar{r}_{it}} \right) \left( \frac{1 + w_{it} \bar{R}_t}{1 + \bar{w}_{it} \bar{R}_t} \right) \right]^{1/N}$ .

15

18. A computer readable medium which stores code for programming a processor to perform a geometric performance attribution computation to determine portfolio performance, relative to a benchmark, over multiple time periods  $t$ , where  $t$  varies from 1 to  $T$ , by determining attribution effects  $1 + Q_{ijt}^G$  given by

20 
$$1 + Q_{ijt}^G = \prod_k \left( \frac{1 + a_{ijt}^k}{1 + b_{ijt}^k} \right) \Gamma_{ijt}^k,$$



where  $\Gamma_{ijt}^k$  are corrective terms that satisfy the constraint  $\prod_{ij} (1 + Q_{ijt}^G) = \frac{1 + R_t}{1 + \bar{R}_t}$ , each of  $a_{ijt}^k$  and  $b_{ijt}^k$  is a coefficient for attribution effect  $j$ , sector  $i$ , and period  $t$ ,  $R_t$  is a portfolio return for period  $t$ , the coefficients  $a_{ijt}^k$  and  $b_{ijt}^k$  are obtained from arithmetic attribution effects  $Q_{ijt}^A = \sum_k a_{ijt}^k - \sum_k b_{ijt}^k$  which correspond to the attribution effects  $1 + Q_{ijt}^G$ ,  $\bar{R}_t$  is a

- 5 benchmark return for period  $t$ ,  $R$  is determined by  $R = [\prod_{t=1}^T (1 + R_t)] - 1$ , and  $\bar{R}$  is determined by  $\bar{R} = [\prod_{t=1}^T (1 + \bar{R}_t)] - 1$ , and determining the portfolio performance as

$$\frac{1 + R}{1 + \bar{R}} = \prod_{t=1}^T \prod_{i=1}^N \prod_{j=1}^M (1 + Q_{ijt}^G).$$

19. The computer readable medium of claim 18, wherein  $M = 2$ ,  $1 + Q_{it}^G$  are

- 10 attribution effects for issue election given by  $1 + Q_{it}^G = \frac{1 + w_{it} r_{it}}{1 + w_{it} \bar{r}_{it}} \Gamma_{it}^I$ , and  $1 + Q_{i2t}^G$  are

attribution effects for sector selection given by  $1 + Q_{i2t}^G = \left( \frac{1 + w_{it} \bar{r}_{it}}{1 + \bar{w}_{it} \bar{r}_{it}} \right) \left( \frac{1 + \bar{w}_{it} \bar{R}_t}{1 + w_{it} \bar{R}_t} \right) \Gamma_{it}^S$ ,

where  $r_{it}$  is a portfolio return for sector  $i$  for period  $t$ ,  $\bar{r}_{it}$  is a benchmark return for sector  $i$  for period  $t$ ,  $w_{it}$  is a weight for  $r_{it}$ ,  $\bar{w}_{it}$  is a weight for  $\bar{r}_{it}$ , the values of  $\Gamma_{it}^I$  are

$$\Gamma_{it}^I = \left[ \frac{1 + R_t}{1 + \bar{R}_t} \prod_{i=1}^N \left( \frac{1 + w_{it} \bar{r}_{it}}{1 + w_{it} r_{it}} \right) \right]^{1/N}, \text{ and}$$

- 15 the values of  $\Gamma_{it}^S$  are  $\Gamma_{it}^S = \left[ \frac{1 + \bar{R}_t}{1 + \bar{R}_t} \prod_{i=1}^N \left( \frac{1 + \bar{w}_{it} \bar{r}_{it}}{1 + w_{it} \bar{r}_{it}} \right) \left( \frac{1 + w_{it} \bar{R}_t}{1 + \bar{w}_{it} \bar{R}_t} \right) \right]^{1/N}$ .